A little E&M review

- Coulomb’s Law
- Gauss’s Law
- Ohm’s Law

Coulomb’s Law

Interaction of two point charges, electrostatic attraction between two point charges (of opposite sign)

\[ F = \frac{1}{4\pi\varepsilon}(q_1q_2)/r^2 \]

If \( F \) is divided by \( q \), the Electric Field is defined.

\[ \frac{F}{q} = \frac{1}{4\pi\varepsilon}(q_2)/r^2 \]

\[ E = \frac{1}{4\pi\varepsilon}(q)r^{-2} \]

E field around point charge of value \( q \).

Gauss’s Law: The total electric flux through a closed surface is equal to the total net charge within the surface. In other words, if a closed surface of any shape is constructed in a region in which an E field is present, the surface integral of the normal component of the E field over the surface is equal to the net charge enclosed by the surface divided by the permittivity.

\[ \oint E \cdot \hat{n} dA = \frac{Q}{\varepsilon_0} \]

Gauss’s Law in differential form is

\[ \nabla \cdot E = \frac{\rho}{\varepsilon_0} \]

And in one dimension...

\[ \frac{\partial E}{\partial z} = \frac{\rho}{\varepsilon_0} \]

Charge density may be inferred from the vertical gradient of \( E \).
Ohm’s Law

\[ J = \sigma E \]

\( J \) is the current density, coulombs/m²

For the Earth’s atmosphere, \( J \) is 1.5 \times 10^{-12} \text{ A m}^{-2}

Here the symbol \( \sigma \) is the conductivity with units of S/m, Siemens/meter, or Mhos

Conductivity is a measure of the ease which charge moves through a given medium.

Ohm’s Law may also be expressed in simple “circuit” form, \( V=IR \) where \( V \) is

The voltage, \( I \) is the current and \( R \) is the resistance (Ohms)

What is the value of the conductivity near the Earth’s surface?

\[ \sigma = \frac{J}{E} = \frac{1.5 \times 10^{-12}}{1.5 \times 10^4} = 10^{-14} \text{ Mhos} \]

This conductivity is equivalent to the conductivity of porcelain, which is used as an an insulator. Sigma is sometimes replaced with the symbol \( \sigma \).

Voltage difference is approximately 300 kV

Electrosphere, 25-60 km altitude

Inner circle is Earth, radius of 6370 km

So we live in a giant capacitor. Earth’s surface is the lower plate and the electrosphere about 25-60 km overhead is the upper plate. The voltage difference across the capacitor is about 300 kV. What is the voltage difference across an individual 2 meters tall standing at the Earth’s surface? Upper plate is positive and the lower plate is negative.
The capacitance \( C \) of a concentric spherical capacitor is given by:

\[
C = 4\pi \varepsilon_0 \left( \frac{r_2}{r_2 - r_1} \right)
\]

So what is the capacitance of the Earth-electrode spherical capacitor?

\[
C = 4\pi \times 8.85 \times 10^{-12} \times 6.37 \times 6.395 \times 10^{12} / 2.5 \times 10^4
\]

\( C = 0.18 \) Farads

Measurements of the charge density on the Earth’s surface give a value of \( 3.2 \times 10^{-9} \) Cm\(^2\).

So the total charge on the Earth’s surface is \( 3.2 \times 10^{-9} \) Cm\(^2\) x Area of Earth = 1.65 \( \times 10^6 \) C

WHAT IS THE ENERGY STORED IN THIS CAPACITOR?

\( W = \frac{Q^2}{2C} \)

Plugging in numbers gives \( 7 \times 10^{12} \) Joules

This is the energy of a million 12 V 100 Amp-hr automobile storage batteries.

A little history

- About 1860, Lord Kelvin developed a theory for the global circuit; this was about 40 years before Kennelly and Heaviside postulated the ionosphere (Marconi verified the “Heaviside” layer by being the first to carry our trans-Atlantic radio communications.)
- Lord Kelvin’s work motivated surface measurements of the fair weather electric field, including the Carnegie and Maude cruises. His work also verified that the Earth’s surface carries net negative charge.
- Early 1900’s, CTR Wilson measured E field changes associated with thunderstorms and determined that thunderstorms systematically have positive charge in their upper regions and negative charge in their lower regions. He also proposed that thunderstorms are batteries driving the global circuit. He also suggested that “shower” clouds contribute to the global circuit by transporting negative charge downward on precipitation particles.
We have seen that the Earth's concentric spherical capacitor is a "leaky" dielectric, that is the conductivity of the atmosphere is finite. Without the action of TRW's that transfer net positive charge to the upper electrode and lower net negative charge to the lower electrode, the fair weather electric field would run down over time. We can estimate the e-folding time for the fair weather electric field.

\[ E(t) = E_0 e^{-t/RC} \]

Here RC is the so-called time constant, the product of the atmosphere's resistance and its capacitance.

Combining the following equations......

\[ E \cdot A = \frac{Q}{\varepsilon} \]

\[ C = \frac{Q}{\Delta U} \]

\[ J = \sigma E \]

\[ J = \sigma E A \]

Therefore

\[ RC = \frac{\Delta U C}{\sigma E A} = \frac{\Delta U Q}{\sigma E A \Delta U} = \frac{E A c}{\sigma E A} = \varepsilon / \sigma \]

\[ \varepsilon = 8.85 \times 10^{-12} \]

\[ \sigma = 10^{-6} \]

e-folding time is about 15 minutes!!
The current flowing beneath and above the “thunderstorm”
generators for the global circuit are known as Maxwell currents.
The Maxwell current equation below storms is of the form,
\[ J_M = J_{PD} + J_{CV} + J_P + J_L + \frac{\partial E}{\partial t} \]
\[ J_{PD} \] is the point discharge current, i.e. corona; \( J_{CV} \) is the
convection current; \( J_P \) is the precipitation current; \( J_L \) is the
lightning current; \( J_E \) is the conduction current and the partial
derivative term is the displacement current (current flow due to a
varying E field).

Above thunderstorms the Maxwell current equation is of the
form;
\[ J_M = J_E + \frac{\partial E}{\partial t} \]
Traditionally \( J_M \) has been measured over storm tops, which is much easier to do
provided a high flying aircraft capable of making E measurements is available.

Maxwell current magnitudes

Â Blakeslee et al. (1989) used the NASA ER-2 capable of
measuring the conduction and displacement currents.
Â They found values on the order of several 10’s of nA m² over
many storms.
Â Using this information and the number of thunderstorms
 globally, the total Maxwell current can be found and related
to current flow in the global circuit.
Â A slight inconsistency was found, suggesting the contribution
 of “shower” clouds to the global circuit.

What is current flow in the global circuit?

Â We can find the current flowing in the global
circuit quite easily.
\[ I = J \cdot 4 \pi R_e^2 \]
\[ I = 1.5 \times 10^{-13} A/m^2 \cdot 4 \pi (6.378 \times 10^6)^2 \]
\[ I = 750.4 \]
Â What is the mean resistance of the atmosphere?