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To: GPM Front Range Pilot Study Team
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Subject: Altitude adjustment of JWD observations

Background
The Joss-Waldvogel Disdrometer (JWD) was designed to estimate the size of the raindrops by converting the 'momentum' of the raindrops hitting the sensor head into raindrop diameter sizes. It is assumed that the raindrops are falling at their sea level terminal velocity when they impact the sensor head. For the observations made at BAO and PLT, the raindrops are falling faster then their sea level terminal velocity which means that the JWD will over-estimate the size of the raindrops hitting the sensor head.

This report describes how the JWD recorded diameter sizes are reduced and how the subsequent estimates of N(D), Z, R, Dm, and Do are also reduced. These adjustments will be made to the observations and these 'Altitude Adjusted' observations will be made available to the group. Therefore, you will not need to make these adjustments, but understand how the original recorded diameter sizes were adjusted.

Big Picture Summary
The JWD observations at BAO and PLT over-estimate the diameter of the raindrops impacting the disdrometer sensor head. Relative to the original diameters recorded by the disdrometer, the altitude adjusted diameters are smaller by 1.88% (a factor of 0.981). Relative to the calculations using the original diameter sizes, the altitude adjusted reflectivity is reduced by about 0.8 dBZ, the rain rate is reduced by 5.5% (a factor of $0.981^3 = 0.945$), and the values of D_m and D_o are reduced by 1.88% (a factor of 0.981).

Detailed Description
The JWD can be broken down into three functional blocks: the transducer, the processor, and the digitizer. The output of the transducer is a pulsed voltage that corresponds to the 'momentum' of the raindrop impacting the sensor head. This voltage corresponds to the voltage induced by the coil moving inside the magnetic field in response to the raindrop hitting the sensor head. This voltage is feed to the processor through the cable connecting the outside sensor head with the inside electronics. The processor converts the quasi-square wave voltage into a square-edged pulsed voltage that is feed to the digitizer. The digitizer converts the amplitude of the square wave into digital values that are transmitted to the data acquisition computer using a COM serial port.

In order to understand how the drops falling faster than their sea level terminal velocities effects the recorded raindrop sizes, we need to focus on the output voltage of the transducer. The transducer voltage is a result of the physics of the raindrop impacting the sensor head and the electro-mechanical response of the sensor. The transducer voltage to diameter size relationship is not provided by the manufacturer, but the manufacture does provide a relationship between the raindrop diameter and the processor output voltage. This relationship combines the physics of the raindrop impacting the transducer and the complete system response of the disdrometer and is given as

$$V_{processor} = 0.94D^{1.47}\,. \quad (1)$$

In order to develop a transducer voltage-to-diameter relationship, Kinnell (1976) used simulated rectangular pulses to derive a relationship between the transducer and processor voltages as

$$V_{processor} = 3.98V_{transducer}^{0.39}\,. \quad (2)$$

By combining (1) and (2), a relationship between the transducer voltage and diameter was developed as

$$V_{transducer} = \left[\frac{0.94D^{1.47}}{3.98}\right]^{\frac{1}{0.39}}$$

$$V_{transducer} = 0.025D^{3.77}\,. \quad (3)$$

This derived relationship between transducer voltage and diameter has an exponent of 3.77. Joss and Waldvogel (1977) report that the exponent in (3) is dependent on diameter and varies between 3.1 and 4.3, and the value of 3.77 determined by Kinnell (1976) is in the middle of the expected range.

In order to account for the increased velocity of the raindrops when the JWD is located above sea level, equation (3) needs to be modified to include the raindrop velocity. While the particular diameter-to-fall speed relationship is not important for this application, assume for the moment that the sea level terminal velocity of the raindrops is expressed by (Atlas and Ulbrich 1977)

$$v_{sea}(D) = 17.67D^{0.67}\,. \quad (4)$$

The transducer voltage in equation (3) can now be re-written as

$$V_{transducer} = \frac{0.025}{17.67}D^{3.1}v_{sea}(D)\,. \quad (5)$$

The terminal velocity of the raindrops increases with altitude above sea level and can be approximated by

$$v_{alt}(D) = v_{sea}(D)\left(\frac{\rho_{sea}}{\rho_{alt}}\right)^{0.45} \quad (6)$$

where ρ_{sea} and ρ_{alt} are the atmospheric density at sea level and at the altitude of interest.

The transducer voltage produced by a raindrop of diameter D when the transducer is at an elevated altitude is expressed

$$V_{transducer}\Big|_{altitude} = \frac{0.025}{17.67}D_{alt}^{3.1}v_{sea}(D_{alt})\left(\frac{\rho_{sea}}{\rho_{alt}}\right)^{0.45}\,. \quad (7)$$

The transducer voltage can be made to be only a function of the diameter using equation (4)

$$V_{transducer}\Big|_{altitude} = 0.025D_{alt}^{3.77}\left(\frac{\rho_{sea}}{\rho_{alt}}\right)^{0.45}\,. \quad (8)$$

The disdrometer estimates a raindrop diameter assuming that the instrument is at sea level using the relationship expressed in equation (3). When the disdrometer is located above sea level, the transducer voltage is a result of raindrops following equation (8). To relate the recorded raindrop diameters with the actual raindrop diameters impacting the disdrometer, the transducer voltage of equations (3) and (8) are set equal to each other yielding the relationship

$$0.025D_{alt}^{3.77}\left(\frac{\rho_{sea}}{\rho_{alt}}\right)^{0.45} = 0.025D_{sea}^{3.77}$$

$$D_{alt} = \frac{D_{sea}}{\left[\left(\frac{\rho_{sea}}{\rho_{alt}}\right)^{0.45}\right]^{\frac{1}{3.77}}}. \quad (10)$$

For BAO, the altitude above sea level is 1600 m and the atmospheric densities are approximately $\rho_{sea} = 1.23$ kg m^3 and $\rho_{sea} = 1.05$ kg m^3. Thus, the altitude adjusted diameters relative to the recorded diameters can be expressed using

$$D_{Alt} = 0.981 D_{recorded}. \quad (11)$$

Impact on Integrated Quantities

The following four equations convert the number of drops detected at each diameter size into the number concentration (N(D)), the rain rate distribution (R(D)), liquid water content distribution (W(D)), and reflectivity distribution (Z(D)).

$$N(D_{alt}) = \frac{m(D_{alt})}{F \cdot t \cdot v_{alt}(D_{alt}) \cdot dD_{alt}} \quad (12)$$

$$R(D_{alt}) = \frac{3.6 \cdot \pi}{6 \cdot 1000 \cdot F \cdot t} m(D_{alt}) \cdot D_{alt}^{3} \quad (13)$$

$$W(D_{alt}) = \frac{\pi}{6 \cdot F \cdot t \cdot v_{alt}(D_{alt})} m(D_{alt}) \cdot D_{alt}^{3} \quad (14)$$

$$Z(D_{alt}) = \frac{1}{F \cdot t \cdot v_{alt}(D_{alt})} m(D_{alt}) \cdot D_{alt}^{6} \quad (15)$$

where F is the surface area of the sensor head (0.05 m^2), t is the time interval, and $m(D_{alt})$ is the number of drops detected for the diameter D_{alt}. The velocity $v_{alt}(D_{alt})$ is the altitude adjusted terminal velocity for the raindrop with diameter D_{alt}. The use of $v_{alt}(D_{alt})$ in these altitude adjusted equations prevents a simple proportionality between these equations and the sea level equations. This is because the differences in velocities between $v_{alt}(D_{alt})$ and $v_{sea}(D_{sea})$ are not a simple scaling factor. While the exact change in moments is not possible to calculate, the approximate changes in the moments are estimated using:

$$\frac{Z(D_{alt})}{Z(D_{sea})} \sim \frac{(0.981)^6}{\left(\frac{\rho_{sea}}{\rho_{alt}}\right)^{0.45}} \sim \frac{(0.981)^6}{1.074} \sim 0.831 \Rightarrow 17\% \text{ reduction} \sim \text{-0.8 dBZ} \quad (16)$$

$$\frac{R(D_{alt})}{R(D_{sea})} \sim (0.981)^3 \sim 0.944 \Rightarrow 5.5\% \text{ reduction} \quad (17)$$

$$\frac{W(D_{alt})}{W(D_{sea})} \sim \frac{(0.981)^3}{\left(\frac{\rho_{sea}}{\rho_{alt}}\right)^{0.45}} \sim \frac{(0.981)^3}{1.074} \sim 0.879 \Rightarrow 12\% \text{ reduction} \quad (18)$$

References

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